

Mark Scheme 4753 January 2007

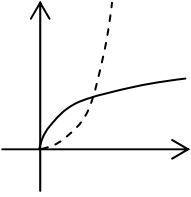
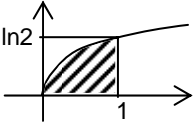
Section A

1 (i) P is (2, 1)	B1	
(ii) $ x = 1\frac{1}{2}$ $\Rightarrow x = (-1\frac{1}{2})$ or $1\frac{1}{2}$ $ x-2 +1 = 1\frac{1}{2} \Rightarrow x-2 = \frac{1}{2}$ $\Rightarrow x = (2\frac{1}{2})$ or $1\frac{1}{2}$	M1 A1 M1 E1	allow $x = 1\frac{1}{2}$ unsupported or $\left 1\frac{1}{2} - 2\right + 1 = \frac{1}{2} + 1 = 1\frac{1}{2}$
or by solving equation directly: $ x-2 +1 = x $ $\Rightarrow 2-x+1 = x$ $\Rightarrow x = 1\frac{1}{2}$ $\Rightarrow y = x = 1\frac{1}{2}$	M1 M1 A1 E1 [4]	equating from graph or listing possible cases
2 $\int_1^2 x^2 \ln x dx$ $u = \ln x$ $dv/dx = x^2 \Rightarrow v = \frac{1}{3}x^3$ $= \left[\frac{1}{3}x^3 \ln x \right]_1^2 - \int_1^2 \frac{1}{3}x^3 \cdot \frac{1}{x} dx$ $= \frac{8}{3} \ln 2 - \int_1^2 \frac{1}{3}x^2 dx$ $= \frac{8}{3} \ln 2 - \left[\frac{1}{9}x^3 \right]_1^2$ $= \frac{8}{3} \ln 2 - \frac{8}{9} + \frac{1}{9}$ $= \frac{8}{3} \ln 2 - \frac{7}{9}$	M1 A1 A1 M1 A1 cao [5]	Parts with $u = \ln x$ $dv/dx = x^2 \Rightarrow v = x^3/3$ $\left[\frac{1}{9}x^3 \right]$ substituting limits o.e. – not $\ln 1$
3 (i) When $t = 0$, $V = 10\,000$ $\Rightarrow 10\,000 = Ae^0 = A$ When $t = 3$, $V = 6000$ $\Rightarrow 6000 = 10\,000 e^{-3k}$ $\Rightarrow -3k = \ln(0.6) = -0.5108\dots$ $\Rightarrow k = 0.17(02\dots)$	M1 A1 M1 M1 A1 [5]	$10\,000 = Ae^0$ $A = 10\,000$ taking lns (correctly) on their exponential equation - not logs unless to base 10 art 0.17 or $-(\ln 0.6)/3$ oe
(ii) $2000 = 10\,000e^{-kt}$ $\Rightarrow -kt = \ln 0.2$ $\Rightarrow t = -\ln 0.2 / k = 9.45$ (years)	M1 A1 [2]	taking lns on correct equation (consistent with their k) allow art 9.5, but not 9.

<p>4 Perfect squares are</p> <p>0, 1, 4, 9, 16, 25, 36, 49, 64, 81 none of which end in a 2, 3, 7 or 8.</p> <p>Generalisation: no perfect squares end in a 2, 3, 7 or 8.</p>	<p>M1 E1</p> <p>B1 [3]</p>	<p>Listing all 1- and 2- digit squares. Condone absence of 0^2, and listing squares of 2 digit nos (i.e. $0^2 - 19^2$)</p> <p>For extending result to include further square numbers.</p>
<p>5 (i) $y = \frac{x^2}{2x+1}$</p> <p>$\Rightarrow \frac{dy}{dx} = \frac{(2x+1)2x - x^2 \cdot 2}{(2x+1)^2}$</p> <p>$= \frac{2x^2 + 2x}{(2x+1)^2} = \frac{2x(x+1)}{(2x+1)^2} *$</p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>E1</p> <p>[4]</p>	<p>Use of quotient rule (or product rule)</p> <p>Correct numerator – condone missing bracket provided it is treated as present</p> <p>Correct denominator</p> <p>www –do not condone missing brackets</p>
<p>(ii) $\frac{dy}{dx} = 0$ when $2x(x+1) = 0$</p> <p>$\Rightarrow x = 0$ or -1</p> <p>$y = 0$ or -1</p>	<p>B1 B1</p> <p>B1 B1</p> <p>[4]</p>	<p>Must be from correct working: SC -1 if denominator = 0</p>
<p>6(i) QA = $3 - y$, PA = $6 - (3 - y) = 3 + y$ By Pythagoras, PA² = OP² + OA²</p> <p>$\Rightarrow (3 + y)^2 = x^2 + 3^2 = x^2 + 9. *$</p>	<p>B1</p> <p>B1</p> <p>E1</p> <p>[3]</p>	<p>must show some working to indicate Pythagoras (e.g. $x^2 + 3^2$)</p>
<p>(ii) Differentiating implicitly:</p> <p>$2(y+3)\frac{dy}{dx} = 2x$</p> <p>$\Rightarrow \frac{dy}{dx} = \frac{x}{y+3} *$</p> <p>-----</p> <p>or $9 + 6y + y^2 = x^2 + 9$</p> <p>$\Rightarrow 6y + y^2 = x^2$</p> <p>$\Rightarrow (6+2y)\frac{dy}{dx} = 2x$</p> <p>$\Rightarrow \frac{dy}{dx} = \frac{x}{y+3}$</p> <p>-----</p> <p>or $y = \sqrt{(x^2+9)} - 3 \Rightarrow \frac{dy}{dx} = \frac{1}{2}(x^2+9)^{-1/2} \cdot 2x$</p> <p>$= \frac{x}{\sqrt{x^2+9}} = \frac{x}{y+3}$</p>	<p>M1</p> <p>E1</p> <p>M1</p> <p>E1</p> <p>M1</p> <p>E1</p>	<p>Allow errors in RHS derivative (but not LHS) - notation should be correct</p> <p>brackets must be used</p> <p>-----</p> <p>Allow errors in RHS derivative (but not LHS) - notation should be correct</p> <p>brackets must be used</p> <p>-----</p> <p>(cao)</p>
<p>(iii) $\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$</p> <p>$= \frac{4}{2+3} \times 2$</p> <p>$= \frac{8}{5}$</p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>[3]</p>	<p>chain rule (soi)</p>

Section B

7(i) When $x = -1$, $y = -1\sqrt{0} = 0$ Domain $x \geq -1$	E1 B1 [2]	Not $y \geq -1$
(ii) $\frac{dy}{dx} = x \cdot \frac{1}{2}(1+x)^{-1/2} + (1+x)^{1/2}$ $= \frac{1}{2}(1+x)^{-1/2}[x + 2(1+x)]$ $= \frac{2+3x}{2\sqrt{1+x}}$ *	B1 B1 M1 E1	$x \cdot \frac{1}{2}(1+x)^{-1/2}$ $\dots + (1+x)^{1/2}$ taking out common factor or common denominator www
<i>or</i> $u = x + 1 \Rightarrow \frac{du}{dx} = 1$ $\Rightarrow y = (u-1)u^{1/2} = u^{3/2} - u^{1/2}$ $\Rightarrow \frac{dy}{du} = \frac{3}{2}u^{1/2} - \frac{1}{2}u^{-1/2}$ $\Rightarrow \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{3}{2}(x+1)^{1/2} - \frac{1}{2}(x+1)^{-1/2}$ $= \frac{1}{2}(x+1)^{-1/2}(3x+3-1)$ $= \frac{2+3x}{2\sqrt{1+x}}$ *	M1 A1 M1 E1 [4]	taking out common factor or common denominator
(iii) $dy/dx = 0$ when $3x + 2 = 0$ $\Rightarrow x = -2/3$ $\Rightarrow y = -\frac{2}{3}\sqrt{\frac{1}{3}}$ Range is $y \geq -\frac{2}{3}\sqrt{\frac{1}{3}}$	M1 A1ca o A1 B1 ft [4]	o.e. not $x \geq -\frac{2}{3}\sqrt{\frac{1}{3}}$ (ft their y value, even if approximate)
(iv) $\int_{-1}^0 x\sqrt{1+x} dx$ let $u = 1 + x$, $du/dx = 1 \Rightarrow du = dx$ when $x = -1$, $u = 0$, when $x = 0$, $u = 1$ $= \int_0^1 (u-1)\sqrt{u} du$ $= \int_0^1 (u^{3/2} - u^{1/2}) du$ *	M1 B1 M1 E1	$du = dx$ or $du/dx = 1$ or $dx/du = 1$ changing limits – allow with no working shown provided limits are present and consistent with dx and du . $(u-1)\sqrt{u}$ www – condone only final brackets missing, otherwise notation must be correct
$= \left[\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2} \right]_0^1$ $= \pm \frac{4}{15}$	B1 B1 M1 A1ca o [8]	$\frac{2}{5}u^{5/2}, -\frac{2}{3}u^{3/2}$ (oe) substituting correct limits (can imply the zero limit) $\pm \frac{4}{15}$ or ± 0.27 or better, not 0.26

<p>8 (i) $f'(x) = 2(e^x - 1)e^x$</p> <p>When $x = 0$, $f'(0) = 0$ When $x = \ln 2$, $f'(\ln 2) = 2(2 - 1)2 = 4$</p>	<p>M1 A1</p> <p>B1dep M1 A1cao [5]</p>	<p>or $f(x) = e^{2x} - 2e^x + 1$ M1 (or $(e^x)^2 - 2e^x + 1$ plus correct deriv of $(e^x)^2$) $\Rightarrow f'(x) = 2e^{2x} - 2e^x$ A1 derivative must be correct, www $e^{\ln 2} = 2$ soi</p>
<p>(ii) $y = (e^x - 1)^2$ $x \leftrightarrow y$ $x = (e^y - 1)^2$ $\Rightarrow \sqrt{x} = e^y - 1$ $\Rightarrow 1 + \sqrt{x} = e^y$ $\Rightarrow y = \ln(1 + \sqrt{x})$</p>	<p>M1</p> <p>M1 E1</p>	<p>reasonable attempt to invert formula</p> <p>taking lns similar scheme of inverting $y = \ln(1 + \sqrt{x})$</p>
<p>or $gf(x) = g((e^x - 1)^2)$ $= \ln(1 + e^x - 1)$ $= x$</p>	<p>M1 M1 E1</p>	<p>constructing gf or fg $\ln(e^x) = x$ or $e^{\ln(1+\sqrt{x})} = 1 + \sqrt{x}$</p>
 <p>Gradient at $(1, \ln 2) = \frac{1}{4}$</p>	<p>B1</p> <p>B1ft [5]</p>	<p>reflection in $y = x$ (must have infinite gradient at origin)</p>
<p>(iii) $\int (e^x - 1)^2 dx = \int (e^{2x} - 2e^x + 1) dx$ $= \frac{1}{2} e^{2x} - 2e^x + x + c$ * $\int_0^{\ln 2} (e^x - 1)^2 dx = \left[\frac{1}{2} e^{2x} - 2e^x + x \right]_0^{\ln 2}$ $= \frac{1}{2} e^{2 \ln 2} - 2e^{\ln 2} + \ln 2 - (\frac{1}{2} - 2)$ $= 2 - 4 + \ln 2 - \frac{1}{2} + 2$ $= \ln 2 - \frac{1}{2}$</p>	<p>M1 E1</p> <p>M1 M1 A1 [5]</p>	<p>expanding brackets (condone e^{x^2})</p> <p>substituting limits $e^{\ln 2} = 2$ used must be exact</p>
<p>(iv)</p>  <p>Area $= 1 \times \ln 2 - (\ln 2 - \frac{1}{2}) = \frac{1}{2}$</p>	<p>M1 B1</p> <p>A1cao [3]</p>	<p>subtracting area in (iii) from rectangle rectangle area $= 1 \times \ln 2$</p> <p>must be supported</p>